Int. J. Heat Mass Transfer. Vol. 28, No. 9, pp. 1637–1643, 1985
Printed in Great Britain

0017–9310/85 $3.00 + 0.00
Pergamon Press Ltd.

Prediction of the heat transfer for decaying turbulent swirl flow in a tube

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(*Received* 12 *August* 1983 *and in final form* 31 *May* 1984)

Abstract—The paper presents an analytical study of the heat transfer characteristics in decaying turbulent swirl flow generated by short twisted-tapes placed at the entrance of the test section. The expressions were obtained by having a series solution of the swirl equation deduced from the Navier–Stokes equations with the aid of an order of magnitude analysis. The approach is found to be in good agreement with the available experimental data, with an average deviation of 5% and a maximum of 15%. An augmentation in heat transfer as high as 80% is obtained.

1. INTRODUCTION

IT HAS BEEN established for more than 60 years that swirling flow will improve heat transfer in duct flow. Swirl flow devices include a number of geometrical arrangements and tube inserts for forced flow that create rotating and/or secondary flow: inlet vortex generators, twisted-tape inserts and axial core inserts with a screw type winding.

Many investigations have been made to determine the heat transfer and pressure drop characteristics for various single-phase fluids in a swirl flow. Among different heat transfer augmentation techniques, Bergles *et al.* [1] reported a bibliography of world literature.

The non-decaying swirl flow had been investigated in the recent past by various investigators [2–9]. The studies [10–12] presented survey and evaluation of the heat transfer data in swirl flow.

A number of correlations have been proposed for the single-phase heat transfer for swirl flow in tubes. Smithberg and Landis [5], Thorsen and Landis [7] and Lopina and Bergles [8] developed analytical expressions for predicting heat transfer in non-decaying swirl flow.

The heat transfer characteristics in decaying swirl flow were investigated by Gambill and Greene [13] who used spiral ramp and tangential slot vortex generators at the inlet of the test section in which water was the working fluid. Blum and Oliver [14] and Migay and Golubev [15] showed that free swirling flow increased the heat transfer rate.

Narezhnyy and Sudarev [16] and Klepper [17] used swirl generators at the pipe inlet in the form of twisted tapes; this produces swirling flow in the heated test-sections and increases the heat transfer. Klepper's study [17] was undertaken to investigate the performance of the swirl flow using nitrogen gas to cool tubes heated by an electric current passing axially through the tube wall with wall-to-gas absolute temperature ratio up to 2.1.

Zaherzadeh and Jagdish [18] reported an experimental study of decaying swirl flow created by tangential vane swirl generated at the inlet of the test section. Hay and West [19] measured the local heat transfer coefficient for air flowing through slot at the inlet of the pipe.

From the foregoing review, it may be inferred that a good deal of published literature is available for heat transfer in axial flow in tubes. However only a little data is available for pressure drop and heat transfer in decaying swirl flow. Further no generalised analytical work seems to have been reported for friction and heat transfer in decaying swirl flow in a pipe. Since such a system offers an interesting phenomenological study of interaction of flow behaviour and heat transfer, the present work [20] has been taken up. It includes the development of an analytical expression for the velocity vector of the decaying flow which in turn gives heat transfer correlation. To test the validity of the approach, the theoretical predictions will be compared with the available experimental data on heat transfer.

2. BASIC EQUATION

Consider a cylindrical coordinate system with x, r and θ as the axial, radial and azimuthal coordinates. Let u, v and w be the time mean velocities in the x, r and θ directions and u', v' and w' be the corresponding turbulent fluctuation velocities. Writing the continuity and Navier–Stokes equations in this notation, taking mean values with respect to time, requiring the mean motion to be steady, the mean density to be constant, density fluctuations to be negligible and symmetry with respect to θ, results in the following governing equations [21]:

$$\frac{\partial}{\partial x}(ru) + \frac{\partial}{\partial r}(rv) = 0 \qquad (1)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial r} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\nabla^2 u - \left[\frac{\partial \overline{u'^2}}{\partial x} + \frac{1}{r}\frac{\partial}{\partial r}(r\,\overline{u'v'})\right] \qquad (2)$$

NOMENCLATURE

a	tube radius
C_p	specific heat at constant pressure
D	tube diameter
$F(y)$	initial condition function, equation (9)
$f(r)$	initial condition function, equation (6)
Gr	Grashof number, $2W_w^2\,Re^2\,\beta\Delta T$
g	acceleration due to gravity
H_a	pitch, tube radii per 360° tape twist
h	heat transfer coefficient
J_0	Bessel's function of the first kind of order zero
J_1	Bessel's function of the first kind of order one
k	thermal conductivity
Pr	Prandtl number, $\mu C_p/k$
Nu	Nusselt number, hD/k
q	rate of heat transfer
Re	Reynolds number, UD/ν
r	radial coordinate
s	total diffusivity factor, $(\nu+\varepsilon)/\nu$
T	temperature
ΔT	wall minus fluid temperature, $T_w - T_b$
U	mean axial velocity in a tube
u	time mean local velocity in X-direction
u'	turbulent fluctuation velocity in X-direction
V_s	resultant swirl velocity, $(U^2+W^2)^{1/2}$
v	time mean local velocity in r-direction
v'	turbulent fluctuation velocity in r-direction
W	non-dimensional local tangential velocity, w/U
w	time mean local velocity in θ-direction
w'	turbulent fluctuation velocity in θ-direction
X	non-dimensional coordinate, x/a
x	axial coordinate
$Y(y)$	function, equation (10)
Y_1	Bessel's function of second kind of order one
y	non-dimensional coordinate, r/a
$Z(z)$	function, equation (10)
z	non-dimensional parameter, DSx/a^2Re.

Greek symbols

α	velocity ratio factor, Vs/U
β	volumetric coefficient of thermal expansion
ε	kinematic eddy viscosity
θ	azimuthal coordinate
λ_n	eigenvalues, $n = 1, 2, 3\ldots$
μ	dynamic viscosity
ν	kinematic viscosity.

Subscripts

b	bulk
cc	centrifugal convection
d	decaying
s	swirl
sc	spiral convection
x	local
w	wall.

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial r} - \frac{w^2}{r} = \frac{-1}{\rho}\frac{\partial p}{\partial r} + \nu\left[\nabla^2 v - \frac{v}{r^2}\right] - \left[\frac{\partial}{\partial x}(\overline{u'vi}) + \frac{1}{r}\frac{\partial}{\partial r}(\overline{rv'^2}) - \frac{\overline{w'^2}}{r}\right] \quad (3)$$

$$u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial r} + \frac{wv}{r} = \nu\left[\nabla^2 w - \frac{w}{r^2}\right] - \left[\frac{\partial}{\partial x}(\overline{u'w'}) + \frac{\partial}{\partial r}(\overline{v'w'}) + 2\frac{\overline{v'w'}}{r}\right]. \quad (4)$$

Where

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}.$$

The resulting set of turbulent Navier–Stokes equations is non-linear and indeterminate due to the turbulent shear terms at the extreme right in equations (2)–(4). Since there are more dependent variables than governing equations, no general method of solution exists. In the absence of general principles on which further mathematical relationships might be based, we are forced to seek an approximate solution by simplifying the equations with the aid of an order-of-magnitude analysis.

3. SOLUTION OF THE SWIRL EQUATION

Equation (4) may be termed as the swirl equation [21]. Since this equation can not be solved as such, it may be simplified [21, 22] by considering the order of magnitude of the various terms in the following manner:

(1) The axial (u) and tangential (w) velocities are much larger than the radial (v) velocity, and the changes with respect to the axial (x) coordinate are smaller than the change with respect to the radial (r) coordinate.

Therefore, in equation (4), the term $v(\partial w/\partial r + w/r)$ can be neglected in comparison with $u(\partial w/\partial x)$.

(2) On the basis of an analysis of the results obtained

by Talbot [22] in decaying laminar swirling flow in a pipe, Kreith and Sonju [21] suggest that the term $v(\partial^2 w/\partial x^2)$ will be negligible compared to $u(\partial w/\partial x)$ in turbulent flow.

(3) Again, on the basis of Laufer's data [23] for fully-developed turbulent flow in a pipe, Kreith and Sonju [21] suggested that on the average the term $\partial(\overline{w'u'})/\partial x$ is an order of magnitude smaller than the term $u(\partial w/\partial x)$. It can be explained further by writing [24] it as

$$-\frac{\mu'}{\rho}\left(\frac{\partial w}{\partial x}+\frac{1}{r}\frac{\partial u}{\partial \theta}\right),$$

where μ'/ρ would correspond to turbulent flow conditions. Because of the assumed symmetry in the θ-direction, $\partial u/\partial\theta = 0$ and the remaining term, $(\mu'/\rho)(\partial^2 w/\partial x^2)$, would be of quite a low order of magnitude in comparison with $u(\partial w/\partial x)$ and hence can be neglected.

(4) The Reynolds stress $\rho\overline{v'w'}$ can be expressed [24] by

$$-\mu'\left[r\frac{\partial}{\partial r}(w/r)+\frac{1}{r}\frac{\partial v}{\partial \theta}\right].$$

Again, due to the assumed symmetry with respect to θ, $\partial/\partial\theta = 0$ and, therefore the term $\partial(\overline{v'w'})/\partial r + 2(\overline{v'w'}/r)$ can be expressed equal to

$$-\varepsilon\left[\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}-w/r^2\right].$$

Where μ'/ρ is replaced by ε, the so-called eddy or turbulent diffusivity.

(5) Substituting $U+\Delta U$ for u in equation (4) yields terms of the form $U(\partial w/\partial x)$ and $\Delta U(\partial w/\partial x)$, where U is the mean axial velocity in fully developed pipe flow. An order-of-magnitude analysis shows that the latter terms are negligible compared to the former.

Based on the simplification outlined above, the swirl equation reduces to the form

$$U\frac{\partial w}{\partial x} = (v+\varepsilon)\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}-\frac{w}{r^2}\right). \quad (5)$$

The boundary and initial conditions on equation (5) are

$$\begin{aligned} w &= 0 \quad \text{at} \quad r = 0 \\ & \qquad\qquad\quad r = a \\ w(r,0) &= f(r) \quad \text{at} \quad x = 0. \end{aligned} \quad (6)$$

Equation (5) is linear and a solution by method of separation of variables [21] is feasible. However, before carrying out the detailed steps of this solution, it will be convenient to non-dimensionalize the equation through the use of the variables.

$$y = \frac{r}{a}, \quad W = \frac{w}{U}, \quad Re = \frac{UD}{v}, \qquad S = (v+\varepsilon)/v, \quad z = \frac{SDx}{a^2 Re}. \quad (7)$$

Equation (5) then takes the form

$$\frac{\partial W}{\partial z} = \frac{\partial^2 W}{\partial y^2}+\frac{1}{y}\frac{\partial W}{\partial y}-\frac{W}{y^2} \quad (8)$$

with boundary and initial conditions

$$\begin{aligned} W(0,z) &= W(1,z) = 0 \\ W(y,0) &= F(y). \end{aligned} \quad (9)$$

Assuming a solution [21] of the form

$$W(y,z) = Y(y)Z(z) \quad (10)$$

and substituting this expression into equation (8) one obtains after rearranging

$$\frac{\dfrac{d^2Y}{dy^2}+\dfrac{1}{y}\dfrac{dY}{dy}-\dfrac{1}{y^2}Y}{Y} = \frac{1}{Z}\frac{dZ}{dz} = -\lambda_n^2 \quad (11)$$

where λ_n^2 are the eigenvalues. The eigenvalues are taken as positive and the minus sign must therefore be used in order to satisfy the physical problem.

The RHS of equation (11) implies that

$$\frac{dZ}{dz}+\lambda_n^2 Z = 0 \quad (12)$$

with a solution

$$Z = An\exp(-\lambda_n^2 z), \quad (13)$$

where An is an arbitrary constant.

The y-dependent part of (11) is

$$\frac{d^2Y}{dy^2}+\frac{1}{y}\frac{dY}{dy}+\left(\lambda_n^2-\frac{1}{y^2}\right)Y = 0. \quad (14)$$

The solution is given [25] in terms of Bessel function by

$$Y(y) = B_n J_1(\lambda_n y)+D_n Y_1(\lambda_n y) \quad (15)$$

and the transformed boundary conditions are

$$Y(0) = Y(1) = 0.$$

The first boundary condition implies $Dn = 0$, and

$$Y(y) = D_n J_1(\lambda_n y).$$

The second boundary condition requires that $J_1(\lambda_n) = 0$. The zeros of $J_P(x_n)$ are well known, and for $P = 1$, the first eight values are

$$\begin{aligned} &\lambda_1^2 = 14.684, \quad \lambda_2^2 = 49.224, \quad \lambda_3^2 = 103.49 \\ &\lambda_4^2 = 177.53 \quad \lambda_5^2 = 271.06, \quad \lambda_6^2 = 384.79 \\ &\lambda_7^2 = 518.02, \quad \lambda_8^2 = 640.4. \end{aligned} \quad (16)$$

The initial condition on $W(y,z)$ is $W(y,0) = F(y)$. To satisfy this initial condition a series solution [21] of the form

$$W(y,z) = \sum_{n=1}^{\infty} C_n J_1(\lambda_n y)\exp(-\lambda_n^2 z) \quad (17)$$

is assumed.

Since the first-order Bessel functions are orthogonal with respect to the weight function y, C_n can be expressed [21] as

$$C_n = \frac{2}{J_0^2(\lambda n)} \int_0^1 yF(y)J_1(\lambda ny)\,dy. \tag{18}$$

The initial condition on $W(y, z)$ is approximated by Kreith and Sonju [21] based on the velocity measurements reported by Smithberg and Landis [5] and is given by the expression

$$W(y,0) = F(y) = H_a^{-1}[6.3y - 0.013(1.1-y)^{-2.68}]. \tag{19}$$

The coefficients C_n, found by numerical integration of equation (18), are

$C_1 = 7.73/H_a$, $C_2 = -5.26/H_a$, $C_3 = 4.04/H_a$,

$C_4 = -3.38/H_a$, $C_5 = 3.0/H_a$, $C_6 = -3.06/H_a$,

$C_7 = 3.33/H_a$, $C_8 = -5.43/H_a$.

From equation (17) the swirl velocity distribution can then be written as

$$\begin{aligned} W(y,z) = {} & \frac{7.73}{H_a} J_1(3.832y)\exp(-14.684z) \\ & - \frac{5.26}{H_a} J_1(7.016y)\exp(-49.224z) \\ & + \frac{4.04}{H_a} J_1(10.173y)\exp(-103.49z) \\ & \pm \cdots \end{aligned} \tag{20}$$

where

$$z = \frac{SDx}{a^2 Re} = \frac{2S}{Re}(x/a) = \frac{2S}{Re}X.$$

For arriving at a useful correlation for predicting heat transfer in decaying swirl flow, it will be proper to decide at this stage about the important parameter, the eddy diffusivity factor S, mentioned in equation (7).

4. EDDY DIFFUSIVITY

The role of eddy diffusivity in prediction of heat transfer in swirling flows is well established. The literature reveals that this parameter is not only a function of r, but the diffusivity is strongly non-isotropic. So a suitable functional relationship for variation of eddy diffusivity is to be selected from the available literature.

The analytical and experimental investigations of the development of incompressible turbulent boundary layers along a concave and a convex stationary annular walls by Yeh [26] represent a valuable contribution to our understanding of the mechanism of turbulent flow. The type of flow studied has applications in some heat transfer equipment where the swirling flow occurs.

An empirical relation by Kreith and Sonju [21] computes a constant diffusivity for solid body rotation in a stationary pipe. Scott and Rask [27] have investigated turbulent viscosities for swirling flow in a stationary annulus. Their method of determining turbulent transport coefficient involves integration and differentiation of large amounts of experimental profiles. They observed that in order to test the validity of their simplified equations, a complete static pressure field, profiles of all six of the Reynolds stresses and a complete mean velocity field for all the three components were needed. Dyban and Epic [28] suggest a method for incorporating the effect of free-stream turbulence on convective heat transfer. The additional eddy viscosity due to turbulence generators can be determined from a minimum of six experimentally-measured turbulence characteristics, i.e. three velocity-fluctuation components, correlation coefficient $\overline{u'v'}$, energy spectrum of the transverse component and the intermittence factor.

It is observed that the value of the eddy diffusivity factor, S, can be expressed in a simple form on the basis of the work of Kreith and Sonju [21]

$$S = 1 + \frac{\varepsilon}{v} = 1 + 4.15 \times 10^{-3}\,Re^{0.86}. \tag{21}$$

It is revealed that away from the wall boundary layers, most of the data by Scott and Rask [27] fall within their prediction. The empirical prediction of Kreith and Sonju represents average value in several senses. First, it applies to both the axial and tangential diffusivities and, second, it does not vary with the radius. It is, therefore, fairly simple to use in the present analysis.

5. HEAT TRANSFER

On the basis of their experiments as well as experience of previous investigators, Lopina and Bergles [8] suggested that the elevated heat transfer in swirl flow can be defined as

$$h_{total} = F(h_{sc} + h_{cc}). \tag{22}$$

For a decaying swirl flow, a similar form may be assumed with $F = 1$,

$$\therefore\; h_{d,x} = h_{sc,x} + h_{cc,x}. \tag{23}$$

The magnitude of the mean tangential component, W_w, at the wall of the tube in decaying swirl flow can be predicted by

$$\frac{w_w}{U} = W_w = \int_0^1 \frac{W(y,z)}{y}\,dy \tag{24}$$

where $W(y, z)$ is given by equation (20).

The velocity increase due to swirling flow can be found by a vector summation of the axial and tangential velocity components, assuming it to be a rotating slug flow. It seems reasonable [8] that the resultant velocity at the tube wall would be of primary interest in predicting h_{sc} and the resultant velocity can be expressed in terms of the axial mean velocity by

$$\alpha = \frac{V_s}{U}$$

where

$$V_s = \sqrt{W_w^2 + U^2} \quad (25)$$

$$\therefore \quad \alpha = (1 + W_w^2)^{1/2}. \quad (26)$$

Using this velocity ratio factor, α, the heat transfer along the axis of the test section can be predicted [8] by

$$h_{sc} = 0.023(\alpha\, Re)^{0.8}\, Pr^{0.4}\,(k/D). \quad (27)$$

The centrifugal convection (h_{cc}) may be predicted [8] by

$$h_{cc} = 0.114(Gr\, Pr)^{1/2}(k/D). \quad (28)$$

The applicable Grashof number is based on the centrifugal acceleration

$$\text{centrifugal acceleration} = \frac{w_w^2}{a}. \quad (29)$$

Combining equation (29) with the free convection Grashof number yields

$$Gr = \frac{D^3\rho^2}{\mu^2}\left(\frac{w_w^2}{a}\right)\beta\Delta T. \quad (30)$$

Using equation (24), $W_w = w_w/U$, we obtain

$$Gr = 2W_w^2\, Re^2\, \beta\Delta T. \quad (31)$$

The simplest and most convenient approach to account for property variations of gases is to evaluate all fluid properties at bulk temperature and to introduce a temperature factor [17] of the form $(T_w/T_b)^{-0.5}$.

Combining this with equations (23), (27) and (28) yields the final prediction equation for heating in decaying swirl flow

$$h_{d,x} = \frac{k}{D}\,[0.023(\alpha\, Re)^{0.8}\, Pr^{0.4} + 0.114(2W_w^2\, Re^2\, \beta\Delta T\, Pr)^{1/2}]\left[\frac{T_w}{T_b}\right]^{-0.5}. \quad (32)$$

6. COMPARISON WITH EXPERIMENTAL DATA

Klepper [17] reported the data of his experimental investigation on the decaying region downstream from the short twisted tapes extended over a portion of a heated length of a tube. Data from this work has been plotted in Figs. 1–3 for comparison and testing the validity of the correlation (32). These results appear to be in good agreement with values calculated based on equation (30). In general the average deviation is about 5% with a maximum of 15%.

7. CONCLUSIONS

The present analytical study predicted the heat transfer characteristics for decaying swirl flow generated by short twisted tapes placed at the entrance of the test section. Expressions were obtained by having a series solution of the swirl equation (5) and the theoretical correlations so obtained can be used for decaying swirl flow in turbulent region.

The theory evolved in respect of heat transfer is in good agreement with the experimental results of Klepper [17]: the average deviation being 5% with a maximum of 15%. The augmentation in local heat transfer is as high as 80%. An initial length of about 60 tube diameters is to be useful from the augmentation point of view.

The results of the theoretical analysis in Figs. 1–3

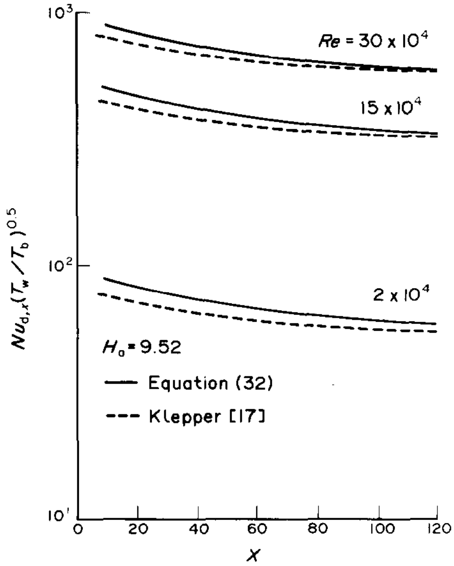

FIG. 1. Heat transfer in decaying swirl flow.

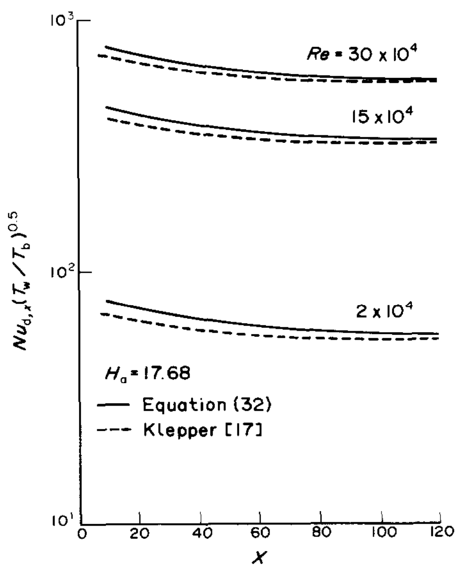

FIG. 2. Heat transfer in decaying swirl flow.

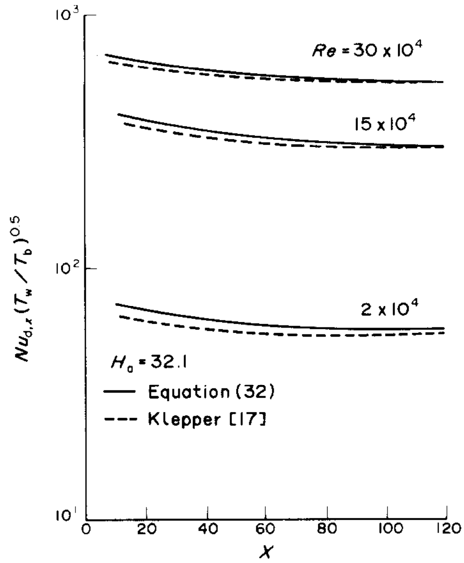

FIG. 3. Heat transfer in decaying swirl flow.

reveal that apart from other parameters, the geometry of the swirl inducer, characterised by H_a, has a marked effect on the heat transfer. Thus it becomes evident that the smaller the value of H_a the bigger the w swirl intensity and hence the higher the heat transfer.

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PREVISION ANALYTIQUE DU TRANSFERT THERMIQUE POUR UN ECOULEMENT DANS UN TUBE, TOURBILLONNAIRE, A TURBULENCE DECROISSANTE

Résumé—On présente une étude analytique du transfert thermique dans un écoulement tourbillonnaire et à turbulence décroissante généré par des rubans spirales et courts placés à l'entrée d'une section d'essai. Les expressions sont obtenues à partir d'une solution série de l'équation de tourbillon déduite des équations de Navier–Stokes, à l'aide d'une analyse des ordres de grandeur. L'approche est trouvée en bon accord avec les données expérimentales disponibles, avec une déviation de 5% et un maximum de 15%. On obtient un accroissement de transfert de chaleur allant jusqu'à 30%.

ANALYTISCHE BERECHNUNG DES WÄRMEÜBERGANGS FÜR ABKLINGENDE TURBULENTE WIRBELSTRÖMUNGEN IN EINEM ROHR

Zusammenfassung—In dem Bericht wird eine analytische Untersuchung des Wärmeübertragungsverhaltens in abklingenden, durch kurze Drallkörper am Eintritt der Versuchsstrecke erzeugten turbulenten Wirbelströmungen vorgestellt. Die Ausdrücke ergeben sich aus einer Reihenlösung der Wirbeltransportgleichung, die mit Hilfe einer Betrachtung der Größenordnungen aus den Navier–Stokes'schen Gleichungen abgeleitet wurde. Es hat sich herausgestellt, daß die Näherung gut mit den verfügbaren experimentellen Daten übereinstimmt, mit einer durchschnittlichen Abweichung von 5% und einer maximalen Abweichung von 15%. Man erhält eine Verbesserung des Wärmeübergangs von 80%.

АНАЛИТИЧЕСКИЙ РАСЧЕТ ТЕПЛООБМЕНА ДЛЯ ЗАТУХАЮЩЕГО ТУРБУЛЕНТНОГО ЗАКРУЧЕННОГО ТЕЧЕНИЯ В ТРУБЕ

Аннотация—В работе представлено аналитическое исследование характеристик теплообмена в затухающем турбулентном течении, закрученном с помощью коротких спрофилированных пластин, помещенных на входе рабочего участка. На основе представленного в виде ряда решения уравнений для закрученного потока, полученных из уравнений Навье–Стокса, а также асимптотического анализа порядка величин различных слагаемых полученного решения найдены соотношения для характеристик теплообмена. Полученные результаты хорошо согласуются с имеющимися экспериментальными данными со среднеквадратичным отклонением в 5% и максимальным—в 15%. Показана возможность увеличения теплообмена до 80%.